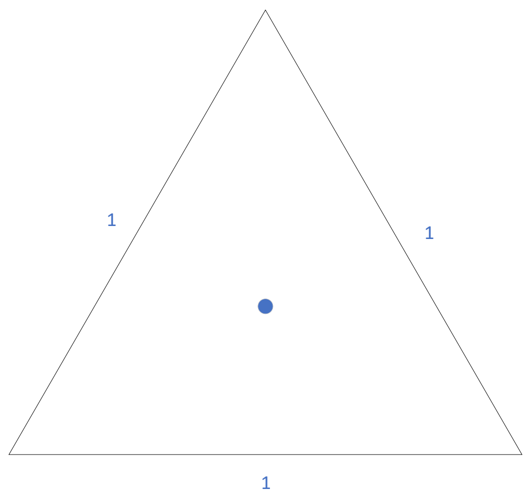
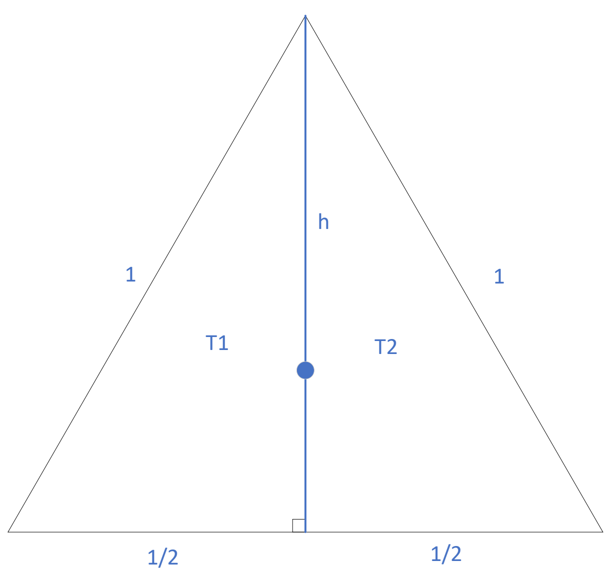
A TRIANGLE OF NO INSIGNIFICANCE

An equal lateral triangle has many secrets hidden within it. Equal implying all the sides being the same length. On of a set of the available triangles. A triangle with 2 sides of same length is an isosceles triangle. Notice that an isosceles triangle is just a subtype of the triangles and that an equilateral triangle is just one type of the isosceles triangles, the last side now just being equal to the other two. Here we are going to have some fun, get some algebra involved and find ways to solve what at first *appears* a simple problem. How to find the center of that most perfect triangle, the equilateral triangle. The goal is simple if you involve trigonometry. But setting that aside what other way may we find it? This article describes one. Here is our triangle, all we need to do is to find the center, intuition would tell us it would be dead center of the triangle:



This is a unit length equilateral triangle. If you think about it, the size of the triangle is not relevant to the solution. What we are after is the distance to the side or the point that goes through such a triangle. Note the lengths of the sides are unit length 1, we may bisect our triangle as shown above into two triangles, let's call them T1 and T2:



Note the triangles are technically the same, just mirror images of one another. We know our center lies somewhere along the line 'h'. But these are right triangles and Pythagoreans theorem may be applied:

$$a^2 + b^2 = c^2$$

Here c is just 1, let's let **a** represent **h** and in this case then **b** would be the bottom of the triangle which is just half of 1 or $\frac{1}{2}$. Our problem resolves to just:

$$h^2 + (\frac{1}{2})^2 = 1^2$$

Taking the square of the constants:

$$h^2 + \frac{1}{4} = 1$$

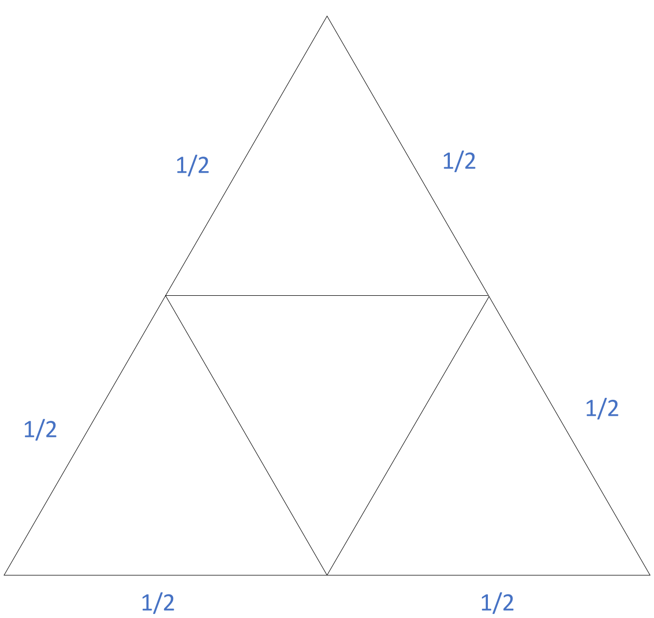
Solving for h^2:

$$h^2 = \frac{3}{4}$$

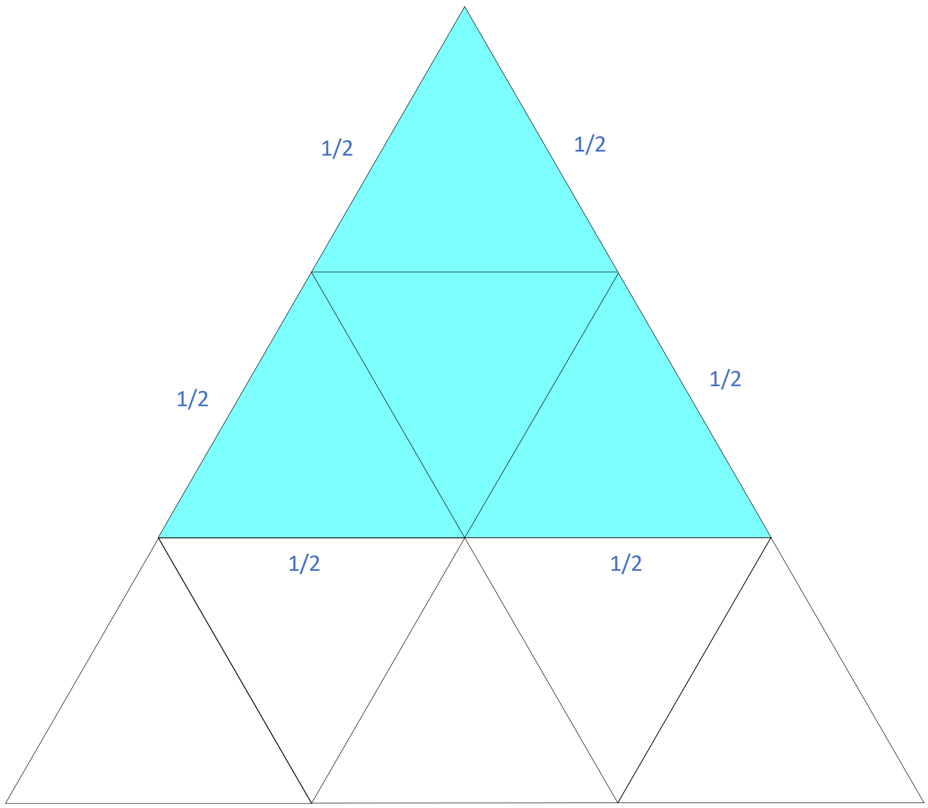
Thus, the distance from the top to the bottom of an equilateral triangle with unit length is:

$$h = \sqrt{\frac{3}{4}}$$

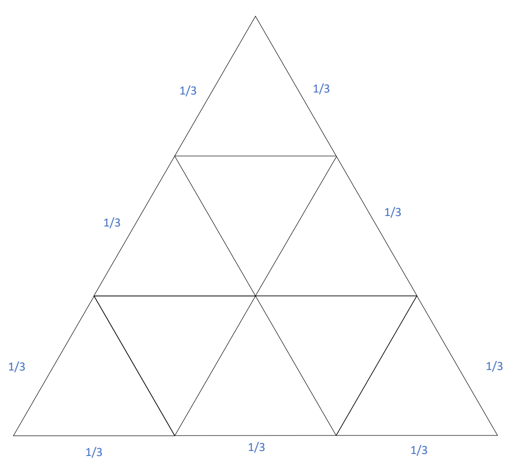
Now an equilateral triangle has a really cool structure. Being that you can compose an equilateral triangle out of other equilateral triangles, such as this:



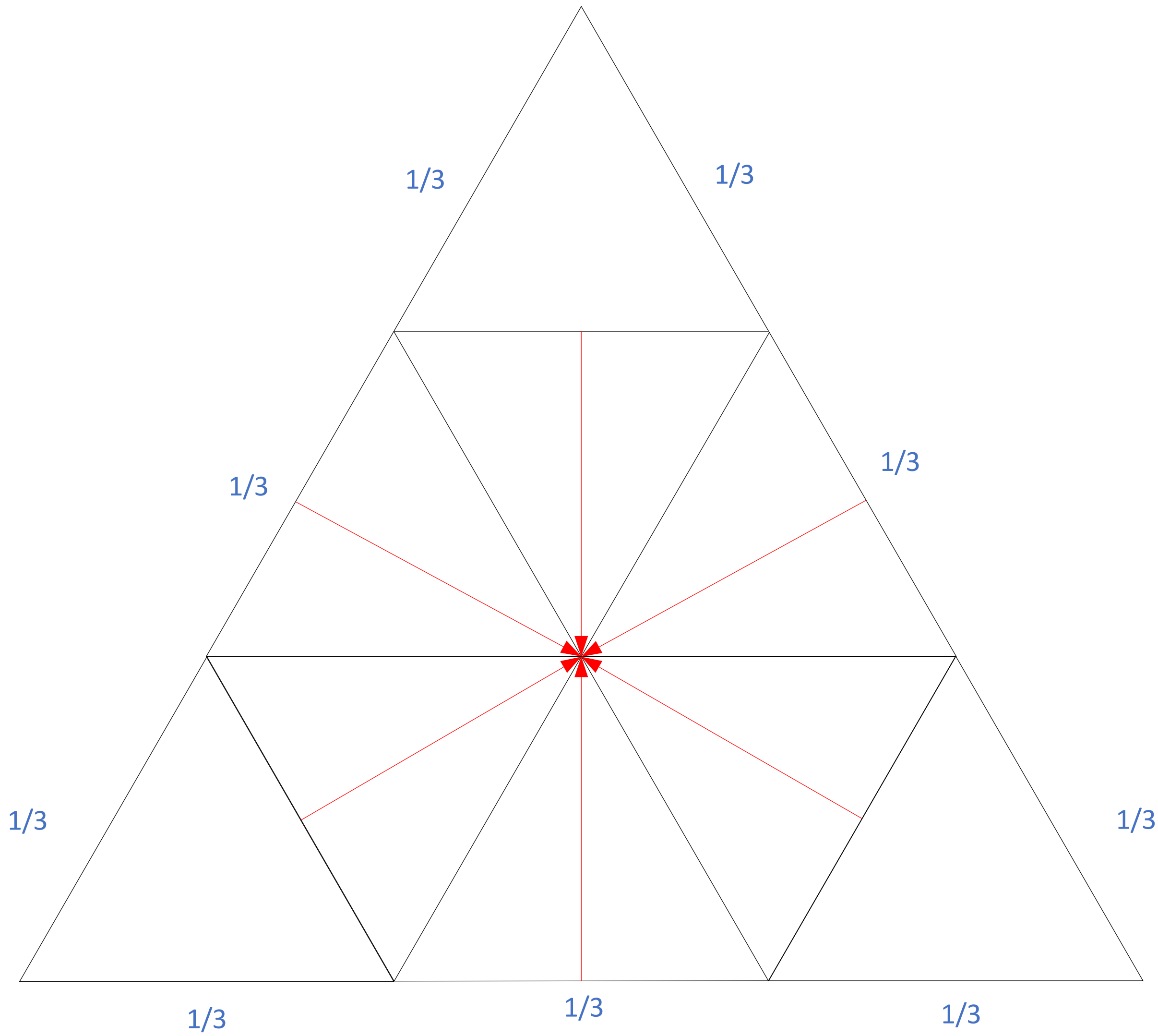
Now our triangle divided into 4 triangles, note the triangles have lengths ½ of the outside triangle. Now let's add four more equilateral triangles to the base of this one:



Our original unit length triangle is the top one, but what if we now make our original unit length triangle this entire new larger equilateral triangle? We end up with a new equilateral triangle, if we allow the sides again to take unit length we get division of 3 on all sides and all triangle edges are 1/3 in length of this larger outside triangle:

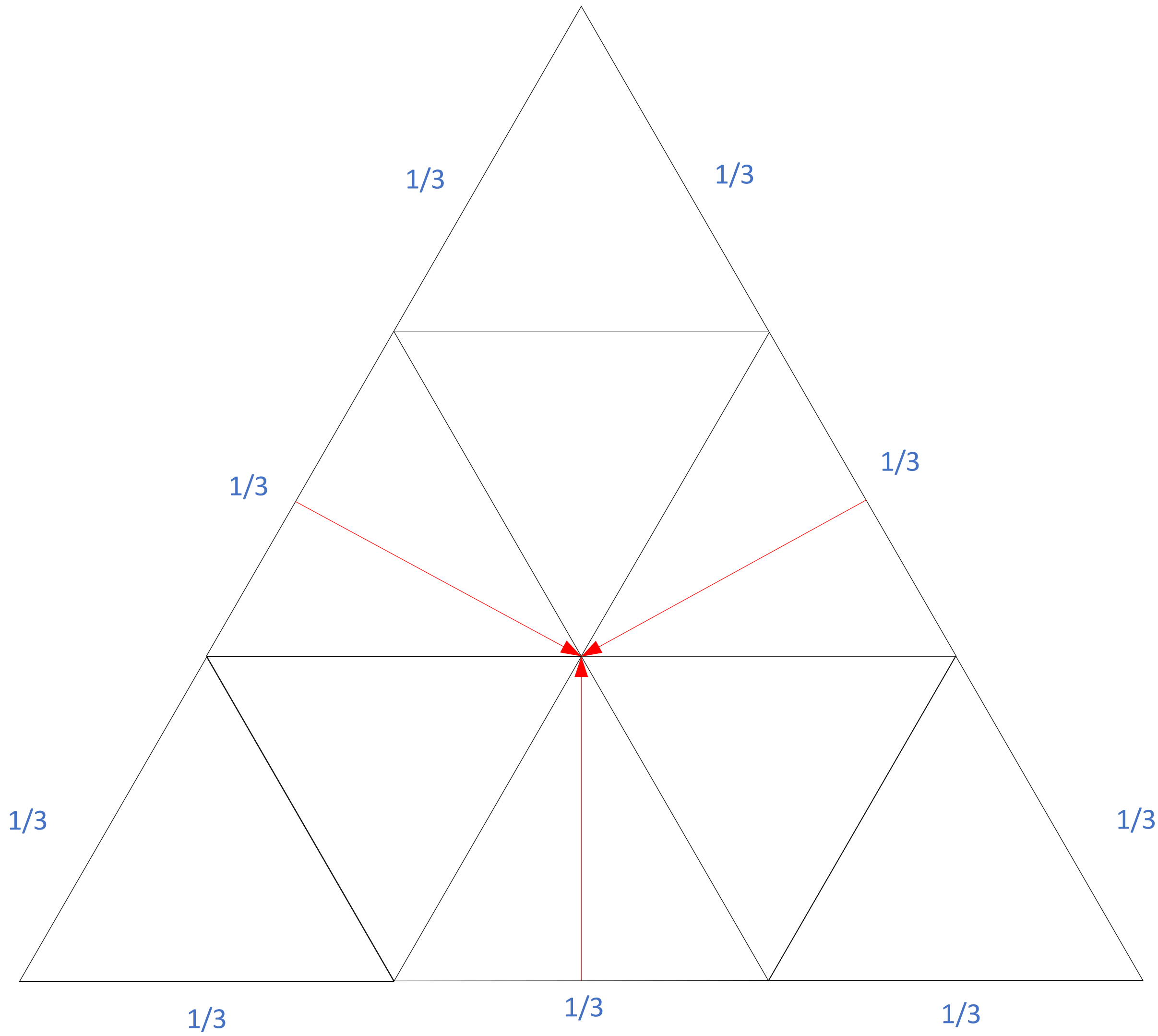


Notice now the answer we seek is being directly pointed at, via the 6 triangles in the center that form a hexagon:



As before: $h = \sqrt{\frac{3}{4}}$, describes the distance for a line segment going from the pinnacle of the triangle to its base and tangent to that base.

Looking at the triangles that are coming from the sides:



We see they are only 1/3 of that unit length from the center if we take their apex to their base with tangent line at base, thus the <u>shortest</u> distance from the base to <u>any</u> side would simply be:

$$h = \frac{1}{3}\sqrt{\frac{3}{4}}$$

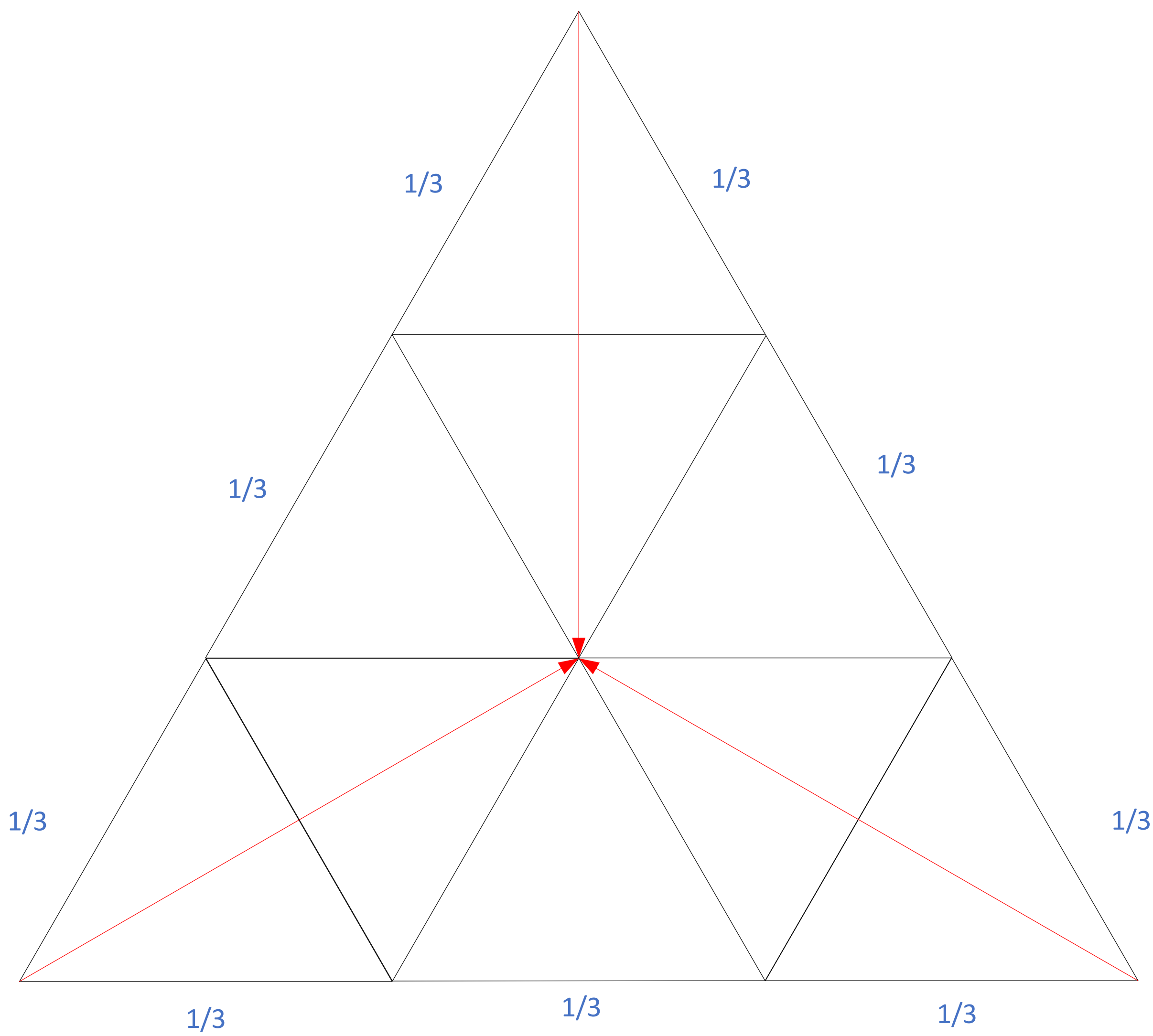
Simplifying:

$$h = \sqrt{\frac{1}{12}}$$

In other words, the shortest distance from the side for any equilateral triangle is nothing more than

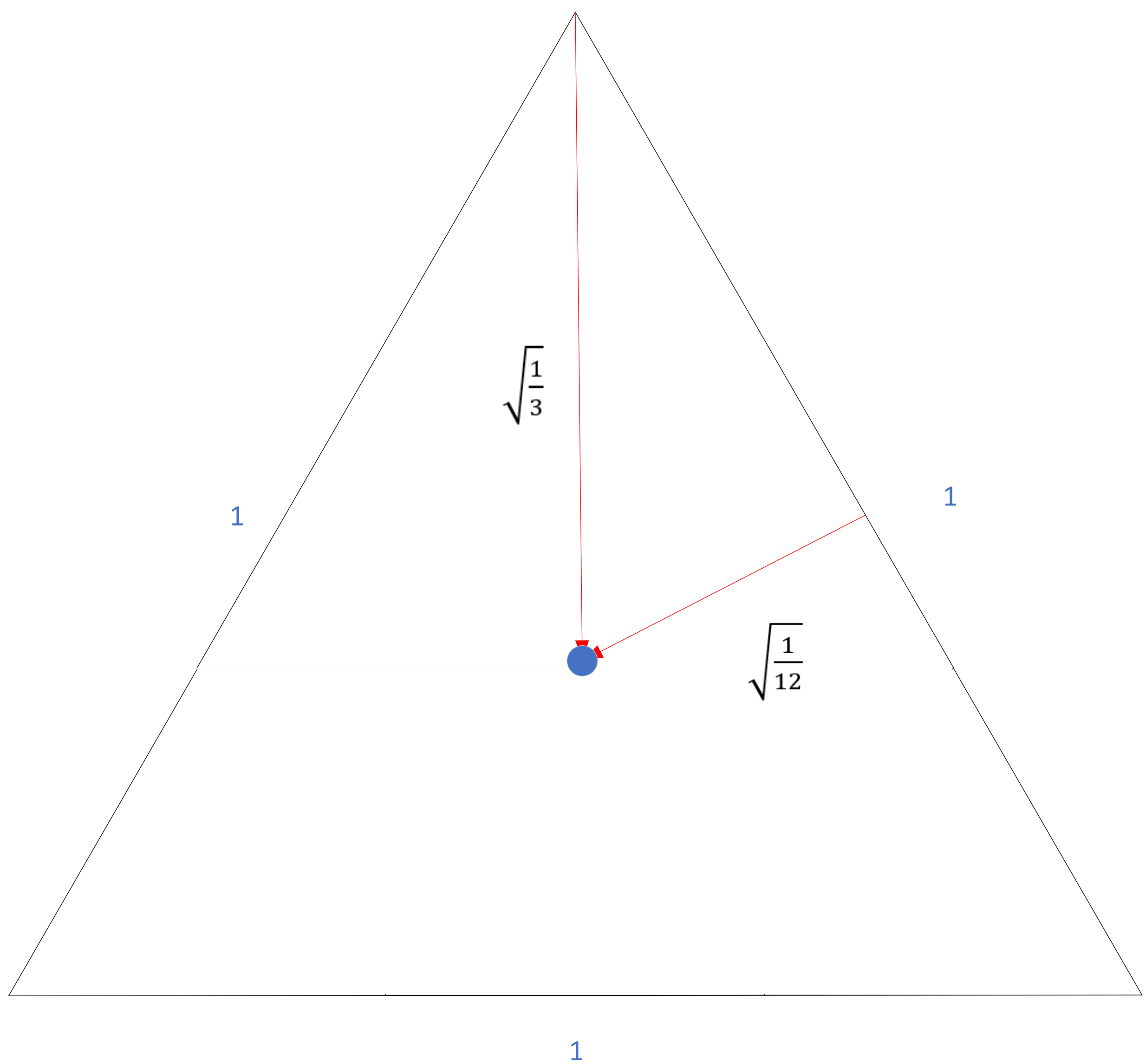
$\sqrt{\frac{1}{12}}$ of the length of its side.

Now the distance from any vertex of the equilateral triangle can be seen as follows, more than twice this amount:



Notice we just passed through 2 such triangles thus the distance is $2\sqrt{\frac{1}{12}}$ or $\sqrt{\frac{1}{3}}$

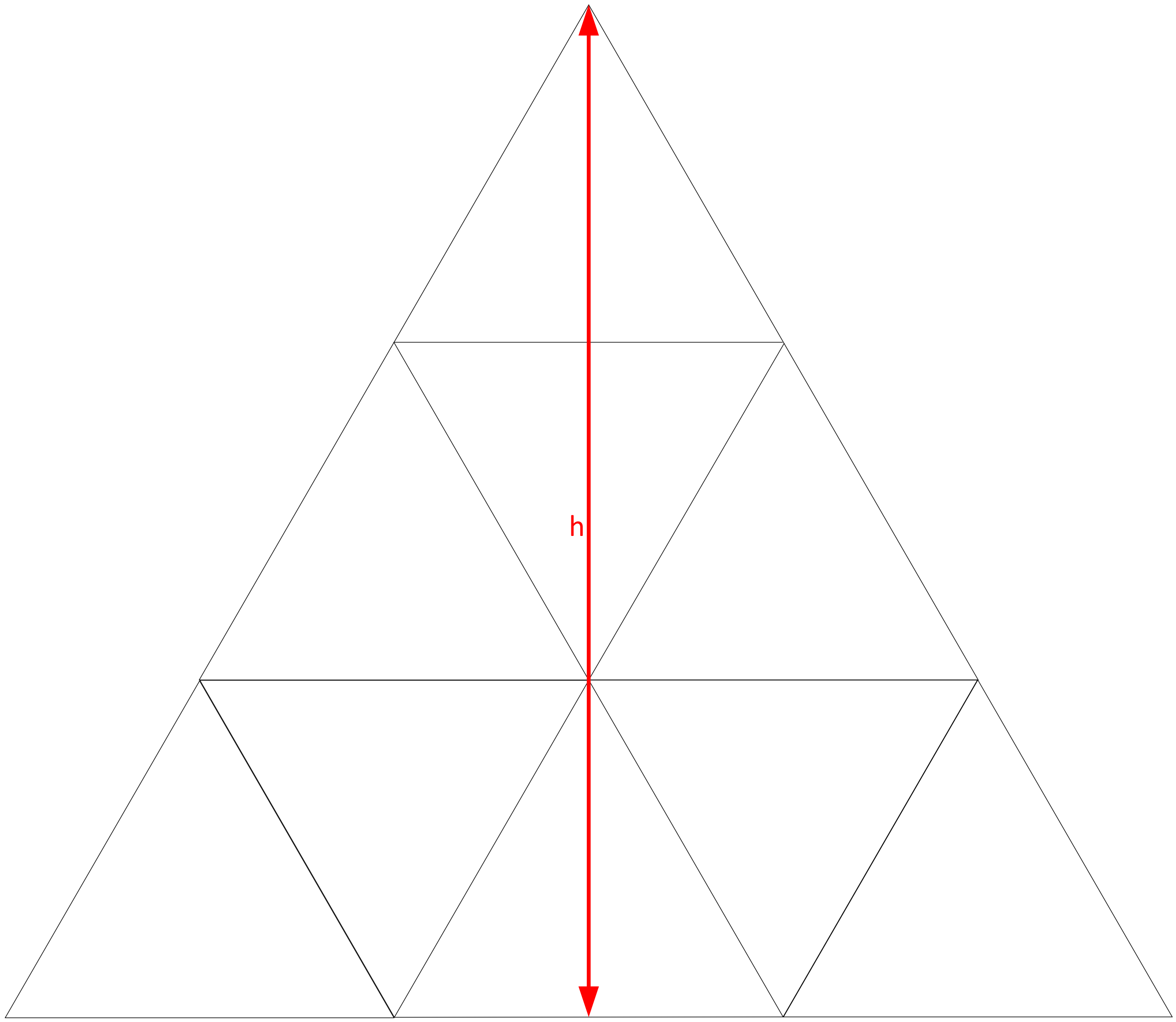
Thus, in the end here it is, all wrapped up, the distance to the center of an equilateral triangle with respect to the length of its sides:



And the proof? Simple remember the distance for h on unit length triangle was:

$$h = \sqrt{\frac{3}{4}}$$

But this passes through dead center of three of our triangles on it’s journey from top to bottom:



Thus, it has to be 3 times the “h” for the smaller triangles:

$$3\sqrt{\frac{1}{12}} = \sqrt{\frac{9}{12}}$$

Dividing the inside fraction by 3 gives us:

$$h = \sqrt{\frac{3}{4}}$$

We have come full circle and proven the original prediction. Thus, with nothing but the Pythagorean theorem and some very simple algebra we arrive at the answers of how to find the center of an Equilateral triangle.

I am certain there are other creative ways to get to the ratios defining the center of an equilateral triangle. If you find any please blog me or email me, I would be interested in other ways to get there. Hope you enjoyed this short narrative on finding the center of an equilateral triangle using nothing but simple mathematics.

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