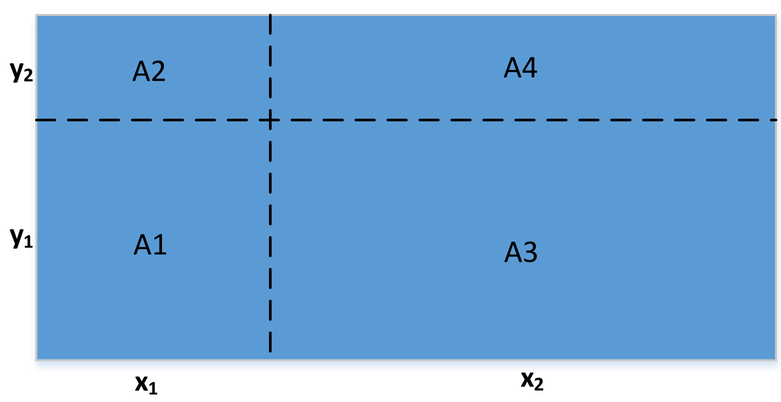
ALGEGRA, GEOMETRY AND THE PYTAGOREAN THEOREM

Algebra, Geometry are intertwined items like two sides of the same coin. Geometric patterns may lead to algebraic expressions and the inverse as well. Take a variable **x**, let it represent a length, square it and out pops a square on the geometric scene. And it's area like nothing more than **x^2**. The abstract must now take on a reality. The **x** length being just a number say 5 squaring it we get 25. In the real world this must represents something, thus without units the number states little about the length of that square nor its area. So assign units, things start to come into focus and the carpenter, mason may now take advantage of it. That unit length being 1=1 Foot. Now the area takes on meaning with length and area popping out for something useful. There it is now the carpenter states, that is 5 Ft by 5 Ft, that is **x^2** and then that area 25 <u>Square</u> Feet pops out. He needs 25 <u>Square</u> feet of lush carpet for that little room.

Thus is the concept of area, now staying to the abstract and taken into account no units needed, generalizations and rules may be attained. Now we have variable **x** and a variable **y**. If **$x=y$** then we just have a square again, but when not, we get a rectangle, one side having length **x** the other length **y** and just like the product of the two resulted in an area in the previous case, thus it is here by no means different. For the area of that rectangle be it whatever the **x** and **y** is its area will be product of the two.

Now comes along a more intriguing question, say that one side that was **x** for that rectangle is divided into two parts. It matters not where the spilt occurs, we end up with two **x** values each denoted in symbolic terms as **x_1** and **x_2** there sum being **$x=x_1+x_2$**. The **y** likewise broken in two with **$y=y_1+y_2$**. Now use substitution. The area of the rectangle being **xy** but now knowing what **x** is the substitution can be applied for **x** is **x_1+x_2**. Ditto, for y and thus **$xy=(x_1+x_2)(y_1+y_2)$** thus a product of two sums. Now the age old geometry steps in to assist. Like we already stated that the product of **xy** is a rectangle, it matters not the values of **x** or **y** and in geometric terms we get this.



With **xy** being the product of the entire area that is **A1+A2+A3+A4**. **A1** is **x_1y_1**, **A2= x_1y_2**, **A3= x_2y_1** and lastly **A4= x_2y_2**. Thus four rectangles create the full sum of the **xy** rectangle and by substitution we get:

$(x_1+x_2)(y_1+y_2)= x_1y_1+ x_1y_2+ x_2y_1+x_2y_2$

Note the combinatorial nature of the terms. With two terms each all 4 possible combinations have been utilized to reach the final area.

y_2	x_1y_2	x_2y_2
y_1	x_1y_1	x_2y_1
	x_1	x_2

Thus matrix algebra came into being. Here a coulomb $[y_1+y_2]_{-T}$ * $[x_1+y_2]$ comes into being and the matrix algebra gives the same answer we already had.

THE FOIL METHOD

Anyone proficient in algebra is well aware of the FOIL method. It involves going to the grocery store and buying some aluminum foil. Yea right☺. The method by which the product of two sums may be acquired:

$(x_1+x_2)(y_1+y_2)$

The technique asserts that the terms be taken from the acronym which stands for First, Outer, Inner, Last. That is to find the terms that create the solution, we take the first two terms of each x_1y_1 add the outer terms of each x_1y_2 then the inner terms of each x_2y_1 and lastly x_2y_2 and thus the result.

$x_1y_1+ x_1y_2+ x_2y_1+x_2y_2$

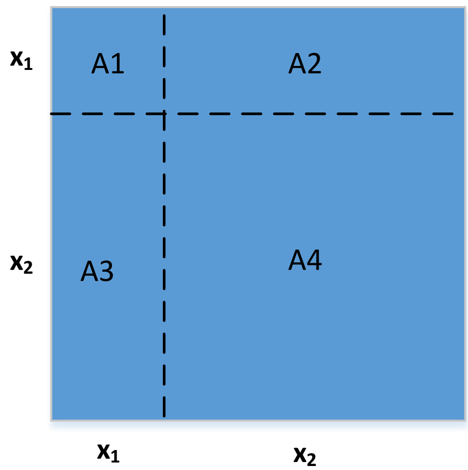
The same answer we got earlier via a mechanical method taught in school that results in the same answer from geometry. Thus the wonderful theorems of the algebra composing of its mechanistic approach to finding answers and the geometric qualities that unify into a single thing, geometry and algebra related like two sides of the same coin.

BACK TO THE SQUARE

When I was a kid I was square in the vernacular of Pennsylvania, boring, dull, not much of a conversationalist, not much a social celebrity. With social skills lacking so it is with the geometric square. Unlike the manifold of shapes and sizes rectangles may take, it fits a unique set much more restricted, much more mundane, its length being nothing more or less than its height. It still being a type of rectangle. Like me being a type of person finding myself inept in social settings. Thus the square sits in the set of the manifold of rectangles, but it stands alone as it cannot be anything more than what it is with its length and width equal, stuck in that situation it remains. That being the case length and width equal that is $x=y$ with area equating to xy, but y is equal to x, thus by substitution: $xy=xx=x^2$. The properties of the rectangle can also be split into parts let $x=x_1+x_2$. This being the case and $x^2=xx$ then the terms come to realization.

$x^2=(x_1+x_2)\ (x_1+x_2)$

Geometry comes along as our amiable companion to explain the riddle and lead us to illumination:



The total area being again just the sum of the rectangles that make up the square A1+A2+A3+A4. But when restrictions are applied as often in society as in mathematics they result in restrictions of the individual. Here the lonely outside square must abide by the rules of geometry just like the rule abiding citizen obeys the rules of the society and like the rule abiding citizen falls within a category he cannot escape, so does the square. Looking at the picture we see two squares have made an appearance on the scene and regardless how large the square may wish to be or how small or lowly. He must abide and out comes two squares. Like two boring adults giving rise to a boring kid: **A1=** x_1^2**, A4=** x_2^2**.** Yet his restrictions carry even further consequences to his demise for **A2=** x_1x_2 and **A3=** x_2x_1**.** Sinking his poor soul into the depths along comes the rules of algebra, multiplication is commutative, implying **ab=ba** thus **A2=** x_1x_2 and **A3=** x_1x_2. Our four terms have re-appeared but restricted to two squares and two basically identical rectangles, one standing tall **A3** and one fallen over on top of the **A4**: **A2**. Being **A2** and **A3** are identical in area, I can resolve to using one symbol to represent them: **B1=A2=A3=** x_1x_2.

Now the terms can be combined for now we have 4 but the last two are just the same:

Area of the square=**A1+B1+B1+A4**

Combining like terms we get:

Area of the square=**A1+2B2+A4**

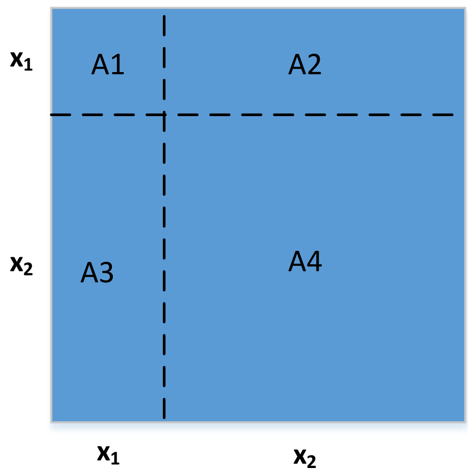
And thrusting ourselves back on the crutch of substitution so easily used in mathematics:

Area of the square is $x_1^2+2x_1x_2+x_2^2$.

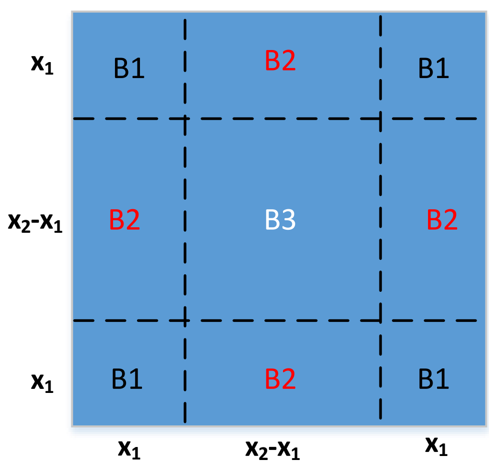
Notice the simplification. Now the boring square blossoms into three terms, two child squares and two identical rectangles, thus is the life of the square.

A MUNDANE SQUARE THAT ARRISES VICTORIES ON THE SCENE

Like many things in life, a mundane thing simple and sweet may give rise to love and tenderness, so it is with our square, for he reveals to use many new things. Singled out among the rectangles he goes on to prove one of the most powerful theorems and the one most used of all. Take our original square nothing to fancy, nothing short of what is was before:



Let all the variables flip away and bisect the square into 9 parts as such:



All we did here is impinge the square composed of $\mathbf{x_1}^2$ and bisect it by allowing the lines of those squares to extend to the ends of the square itself. $\mathbf{x_2}$ was originally the remaining distance after $\mathbf{x_1}$ was subtracted from the total sum of the side being $\mathbf{x}$. Here $\mathbf{x_1}$ has reduced the effective remaining length by itself thus the term $\mathbf{x_2\text{-}x_1}$ in the center. Here we have the following now as symmetry is throughout:

Area of the square = B1+B1+B1+B1+B2+B2+B2+B2+B3

Combining like terms:

Area of the square = 4B1+4B2+B3

The B1 term: $\mathbf{x_1^2}$

The B2 term: $\mathbf{x_1x_2 - x_1^2}$

The B3 term: $\mathbf{(x_2-x_1)(x_2-x_1) = x_2^2-2x_1x_2+x_1^2}$

Using substitution:

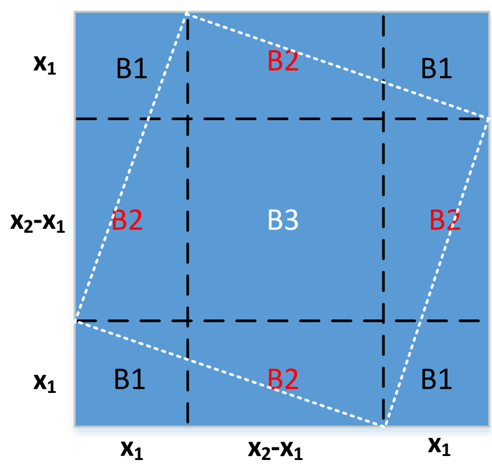
$4x_1^2+4(x_1x_2- x_1^2)+(x_2^2-2x_1x_2+x_1^2)=$

$4x_1^2+4x_1x_2-4x_1^2+ x_2^2-2x_1x_2+x_1^2$

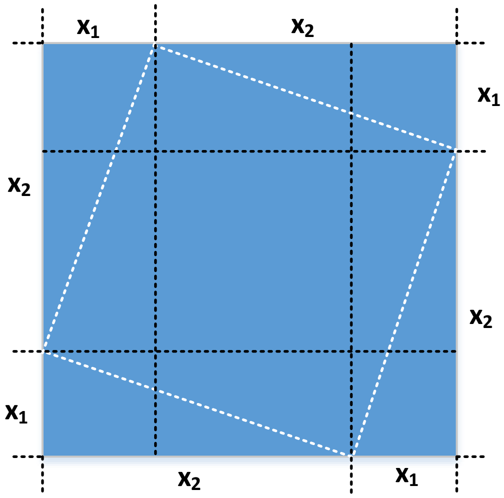
Cancelling like terms and re-arranging:

$x_2^2+4x_1x_2-2x_1x_2+x_1^2= x_1^2+2x_1x_2+x_2^2$

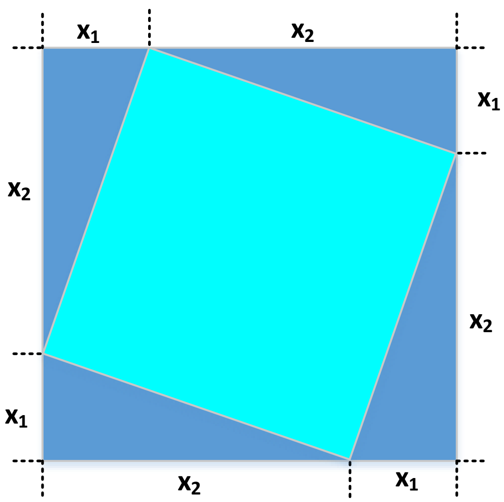
This verifies what we already have known about the square all along and is identical to $\mathbf{x^2=(x_1+x_2)^2}$. Applying the foil method we get $x_1^2+2x_1x_2+x_2^2$. Proving that I have not tried to trick the reader in this bizarre manipulation that I needed to drive home a crucial point being this bisected new square is needed for another trick of the trade which will lead to one of the greatest and well used formulas in mathematics and geometry. Division among friends should be avoided, but here we take it to heart. Let a straight line be drawn starting at the corner of the **B2** square and moving to the opposite corner of the adjacent **B1** Square and repeat this 4 times going around moving 90 degrees with each step, you get the following:



Now for some re-arrangement again. Notice the length of each side of the rectangle that extends from **B1** to **B2** is just $\mathbf{x_1+x_2-x_1= x_2}$. Thus we will replace it with $\mathbf{x_2}$ and remove the black lines from the diagram along with the previous region names:



Now take the area of the rectangles formed by $\mathbf{x_1}$ and $\mathbf{x_2}$ it being $\mathbf{x_1x_2}$ with 4 of them extending around the square. Note if bisected as shown by the white line then the area becomes ½ of that as known when you bisect a rectangle with a line, you divide it into two identical triangles. The blue area now being shown next to clarify it further:



Thus each of the shaded blue areas have an area of $\mathbf{x_1x_2/2}$. Being that there are 4 of them they add up to:

$$\mathbf{4(x_1x_2/2)=2x_1x_2}$$

The remaining area in cyan (that is the inside tilted square) is itself a square within the larger square thus let us give it a value $\mathbf{c^2}$ so the area of our <u>entire</u> square is $\mathbf{c^2+2x_1x_2}$ knowing earlier we proved it remember? The area of this <u>entire</u> square is also expressed in the formula $\mathbf{x_1^2+2x_1x_2+x_2^2}$. Now for the punch line, they are equal so allow that to be so:

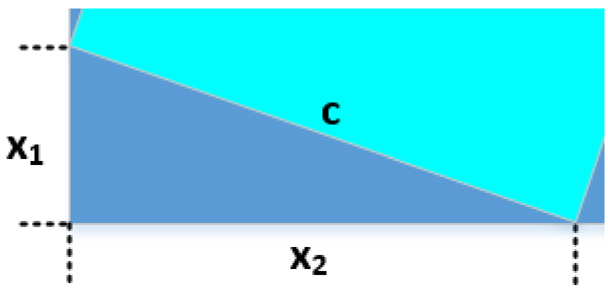
c^2**+2**x_1x_2**=** x_1^2+2x_1x_2+x_2^2

2x_1x_2 is sitting there on both sides thus slips away and we are left with:

c^2**=** x_1^2**+** x_2^2

If c^2 is a true square and it is, then the square root would be the length of a side of that square. The square root of c^2 being the length **c**.

The square and in fact the whole manifold of rectangles included have one thing in common. Every angle is 90 degrees thus the triangle thus formed in the blue area has a 90 degree or what is commonly called a right angle. Here now is one of those triangles, just one of the 4 laid bare for us to see the implications of what has been accomplished:



Typical math books will replace x_1 and x_2 with the symbols **a** and **b** and the formula takes on its canonical form seen in most math text books today as:

c^2**=** a^2**+** b^2

or

$$c = \sqrt{a^2 + b^2}$$

This little formula is the foundation for all trigonometry and geometry stands upon it and is used time and again in all fields from physics, mechanical engineering, to just the average guy having his grass cut lawn with rectangular shape wanting to know how far he must walk to cross it from one corner to the other.

Pythagoras is the man who discovered it and little is known about his life. He was involved in politics, religion, music and mathematics. Being born in 570 BC, he made it to 75 dying in 495 BC. A great Greek Philosopher.

More information about him can be found at: https://en.wikipedia.org/wiki/Pythagoras

Author: Dennis Bingaman

For more information or comments feel free to contact me at:

ddbtech@hughes.net