EXPERIMENTAL RESULTS VS ACTUAL MEASURED DATA FOR THE PENDULUM EXPERIMENT

Not going into the gruesome details of why, the period of a swinging pendulum (that is the time it takes to complete one swing and return to its starting point is an approximation formula that is fairly correct for small angle swings, stated as follows:

$$T \approx 2\pi \sqrt{\frac{L}{g}}$$

Where:

T= the period of swing in seconds

L=the length of the pendulum in meters

G=the gravitation constant on earth of meters/second/second

This formula works well for 'small' angles of swing. For larger angles the Legendre formula must be applied:

$$T = 2\pi \sqrt{\frac{L}{g}} \sum_{n=0}^{\infty} \left(\left(\frac{(2n)!}{(2^n n!)^2} \right)^2 \sin^{2n} \frac{\theta_0}{2} \right)$$

Notice this is the same formula except for a correction factor on the right:

$$\sum_{n=0}^{\infty} \left(\left(\frac{(2n)!}{(2^n n!)^2} \right)^2 \sin^{2n} \frac{\theta_0}{2} \right)$$

The pendulum apparatus can swing a maximum of around 35 degrees from the vertical, though I do not have time to take an infinite sum, I can use Excel to give me an idea of what the correction factor is tending towards after summing so many terms in the infinite sum for 35 degrees here is the result:

n	2nfactiorial	2pwr2*n!	(b/c)^2	sin(L1)^2n	nth term factor	SUM	ANGLE	0.610865238	RADIANS
0	1	1	1	1	1	1	ANGLE	35	DEGREES
1	2	4	0.25	0.090423978	0.022605994	1.022605994			
2	24	64	0.140625	0.008176496	0.00114982	1.023755814			
3	720	2304	0.09765625	0.000739351	7.22023E-05	1.023828016			
4	40320	147456	0.074768066	6.68551E-05	4.99863E-06	1.023833015			
5	3628800	14745600	0.060562134	6.0453E-06	3.66116E-07	1.023833381			
6	479001600	2123366400	0.050889015	5.4664E-07	2.7818E-08	1.023833409			
7	87178291200	4.1618E+11	0.043878794	4.94294E-08	2.1689E-09	1.023833411			
8	2.09228E+13	1.06542E+14	0.038565346	4.4696E-09	1.72372E-10	1.023833411			
9	6.40237E+15	3.45196E+16	0.034399336	4.04159E-10	1.39028E-11	1.023833411			

The SUM is the correction factor after n number of terms

Notice convergence occurs by around the 7^{th} term to 10 decimal places. With our maximum error of 2.38%.

The correction factor drops off to one rather rapidly at smaller angles, here I updated the spread sheet to 20 degrees:

n	2nfactiorial	2pwr2*n!	(b/c)^2	sin(L1)^2n	nth term factor	SUM	ANGLE	0.34906585	RADIANS
0	1	1	1	1	1	1	ANGLE	20	DEGREES
1	2	4	0.25	0.03015369	0.007538422	1.007538422			
2	24	64	0.140625	0.000909245	0.000127863	1.007666285			
3	720	2304	0.09765625	2.74171E-05	2.67745E-06	1.007668962			
4	40320	147456	0.074768066	8.26726E-07	6.18127E-08	1.007669024			
5	3628800	14745600	0.060562134	2.49289E-08	1.50974E-09	1.007669026			
6	479001600	2123366400	0.050889015	7.51697E-10	3.82531E-11	1.007669026			
7	87178291200	4.1618E+11	0.043878794	2.26664E-11	9.94576E-13	1.007669026			
8	2.09228E+13	1.06542E+14	0.038565346	6.83477E-13	2.63585E-14	1.007669026			
9	6.40237E+15	3.45196E+16	0.034399336	2.06093E-14	7.08948E-16	1.007669026			

Note the error is now less than 1%

Pendulum Input data:

Using a tape measure, the distances from the fulcrum to the mirror laser, from the fulcrum to the reference laser and the length of the pendulum were measured:

```
private const double holdingBarWidth=0.0048;   // Meters
private const double laserBeamBreakPoint=0.254;  // Meters
private const double pendulumLength=0.5842;       // Meters
private const double mirrorLaserBeamBreakPoint=0.146; // Meters
```

The holding bar width was measured with a pair of calipers. Being we are using the MKS system, all measurements are converted in meters. These constants are used in the Pendulum class for calculating speeds and angles.

Using the approximation formula, we can calculate the swing period in seconds:

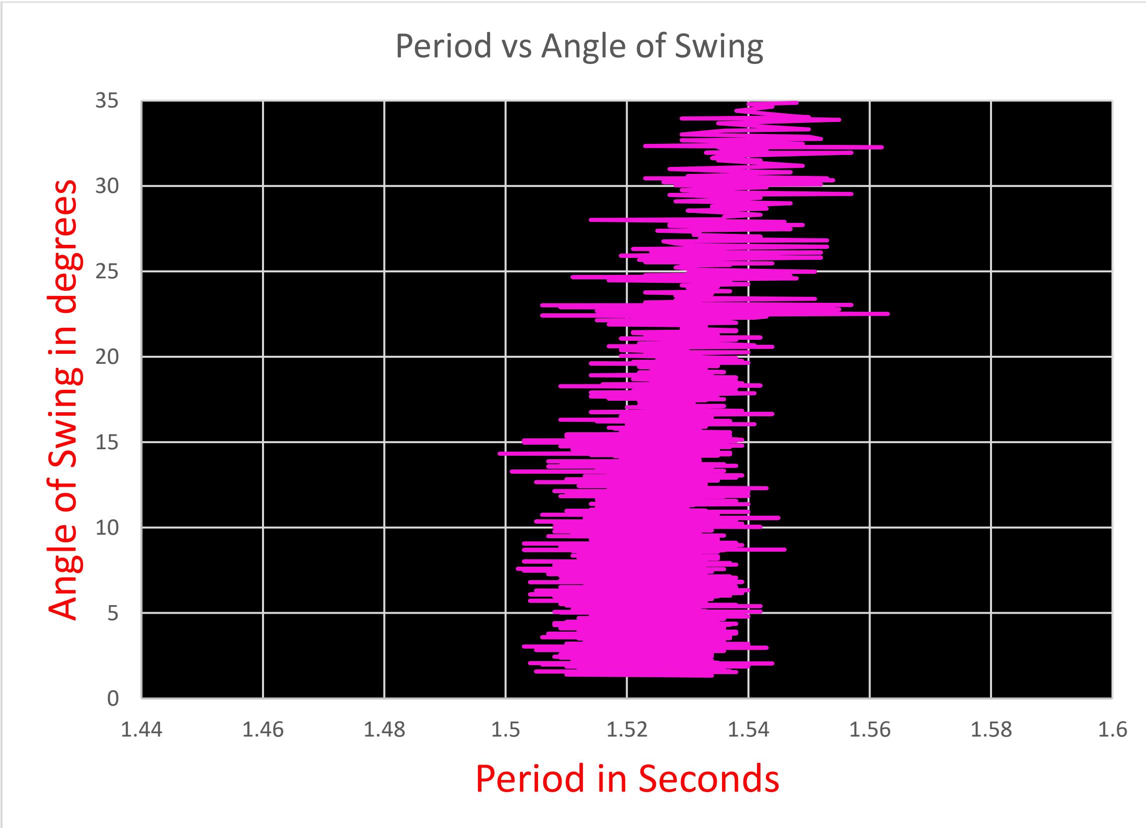
$$T \approx 2\pi\sqrt{\frac{0.5824\ Meters}{9.8\ Meters/sec^2}} = 1.53 \text{ seconds}$$

Taking the average of 927 swings from the raw data collected we get a period of:

COUNT	TIME	PERIOD	SPEED	ANGLE	SP1	SP2	S
913	1392.982	1.512	0.0702	1.682	0.085	0.0889	
914	1394.51	1.528	0.0693	1.66	0.0844	0.0884	
915	1396.028	1.518	0.0684	1.639	0.0838	0.0879	
916	1397.556	1.528	0.0675	1.617	0.0833	0.0873	
917	1399.093	1.537	0.0666	1.595	0.0827	0.0866	
918	1400.598	1.505	0.0656	1.572	0.082	0.0861	
919	1402.136	1.538	0.0646	1.548	0.0813	0.0854	
920	1403.655	1.519	0.0636	1.524	0.0807	0.0847	
921	1405.185	1.53	0.0626	1.499	0.08	0.0841	
922	1406.717	1.532	0.0616	1.474	0.0793	0.0835	
923	1408.246	1.529	0.0605	1.448	0.0786	0.0828	
924	1409.778	1.532	0.0593	1.42	0.078	0.0822	
925	1411.288	1.51	0.0581	1.392	0.0774	0.0816	
926	1412.815	1.527	0.0569	1.362	0.0769	0.081	
927	1414.349	1.534	0.0555	1.33	0.0763	0.0804	
AVERAGE PERIOD:		1.52624026					

Note if we round off, the answer to two decimal places we agree with the approximation formula of 1.53 seconds.

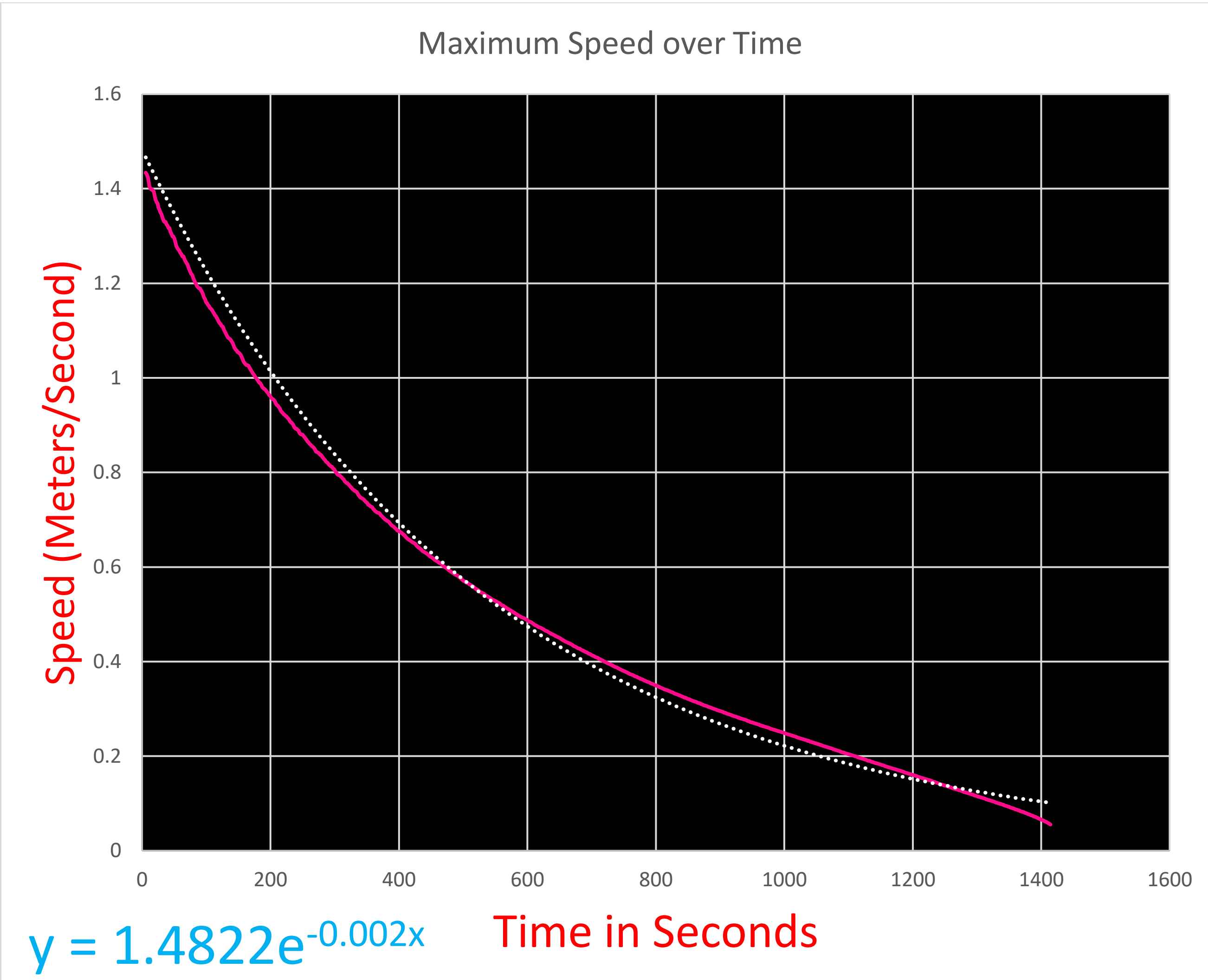
Here is a plot of the Period with respect to the angle of swing, notice the period is fairly constant regardless of the angle of swing as expected as the Period of swing of the pendulum is not affected much by the angle of swing:



Notice the average deviation from 1.53 seconds using the approximation formula is off the larger the angle is. This is in accordance with the Legendre Formula. Using the Legendre Formula, we know we have an error of about 1.02 for 35 degrees effecting the period by 1.53*1.02= 1.56. Though our data is tending towards 1.56 at angles of 35 degrees it never reaches the actual 1.56. The cause of this is unknown to me. Markings on the bottom of the device marked out angles in 5-degree increments. I did notice that the actual angle of swing was less than the calculated angle of swing. The calculated angle of swing is based on the speed that is determined by the interruption time of the primary laser by the Holding Bar Width. My theory is that the measurement of the holding bar width may be off a little and the laser bean width is not accounted for and assumed a straight line, thus generating some errors. From looking at the apparatus and the 5-degree marks, I noticed this data could be off by up to 3-4 degrees (eye-bawling it).

Velocity as a Function of Time

Here is a plot of the maximum pendulum velocity when vertical as a function of time for all swings:



The pink line is the actual data, the white dotted line is an exponential fit, with the exponential function in blue lower left of plot.

Notice the loss in energy (speed) is close to an exponential fall off. I would have expected it to match closer to an exponential fall off than the data it indicating. Frictional loss from swing to swing is apparently a little more complex than just a simple exponential decay with other unknown factors being involved.

The angle of swing can be calculated from the reference laser beam (that is at the vertical position) from the following formula:

$$\emptyset_{in\ degrees} = \frac{180}{\pi} \operatorname{acos}\left(1 - \frac{v^2}{2gl}\right)$$

Where:

$v = speed\ in \frac{meters}{second}$

$g = gravitational\ constant: 9.8\ \frac{meters}{second^2}$

l=length of pendulum in meters

The speed of the pendulum bob (bottom of the pendulum) is calculated as follows:

$$v = \frac{l}{l_{ref}} \frac{w}{p}$$

Where:

$v = speed\ in \frac{meters}{second}$

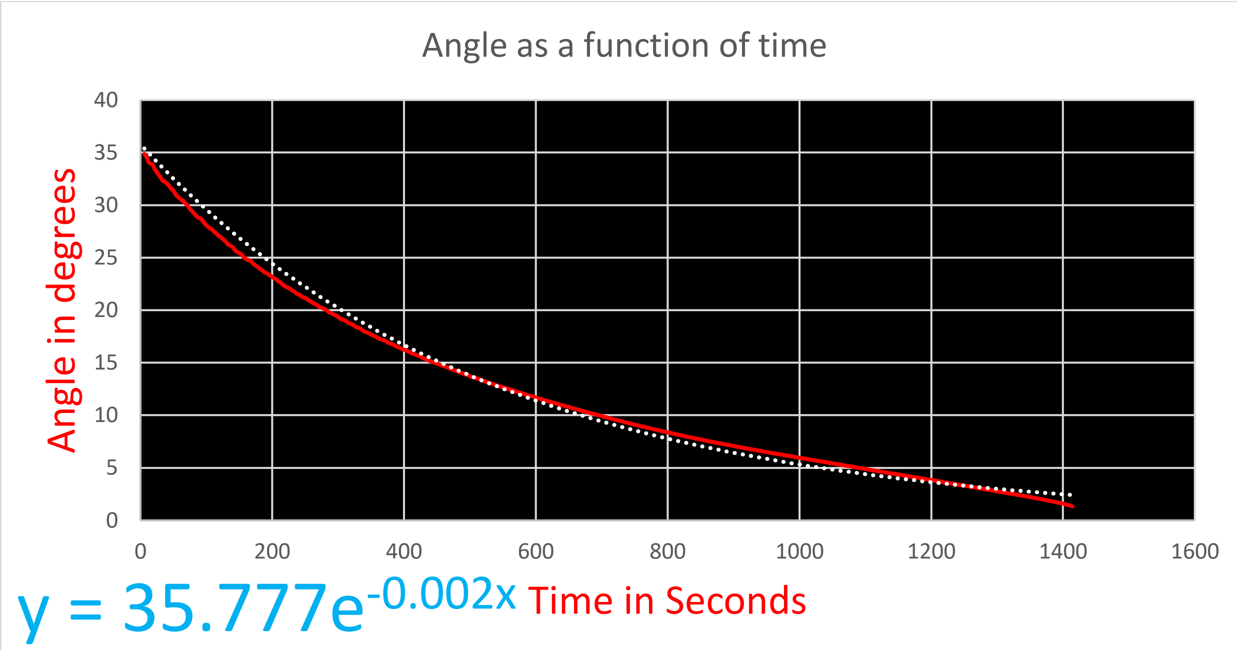
$l = length\ of\ pendum\ to\ bob\ at\ bottom\ in\ meters$

$l_{ref} = length\ of\ pendulum\ to\ ref\ laser\ beam\ to\ mirror\ laser\ beam\ in\ meters$

$w = width\ of\ the\ pendulum\ connecting\ rod\ in\ meters$

$p = time\ the\ laser\ beam\ is\ interrupted\ by\ the\ connecting\ rod\ in\ seconds$

These formulas where used in calculating the speed and the angle of swing data for the pendulum. Here is the result of the angle of swing as a function of time:

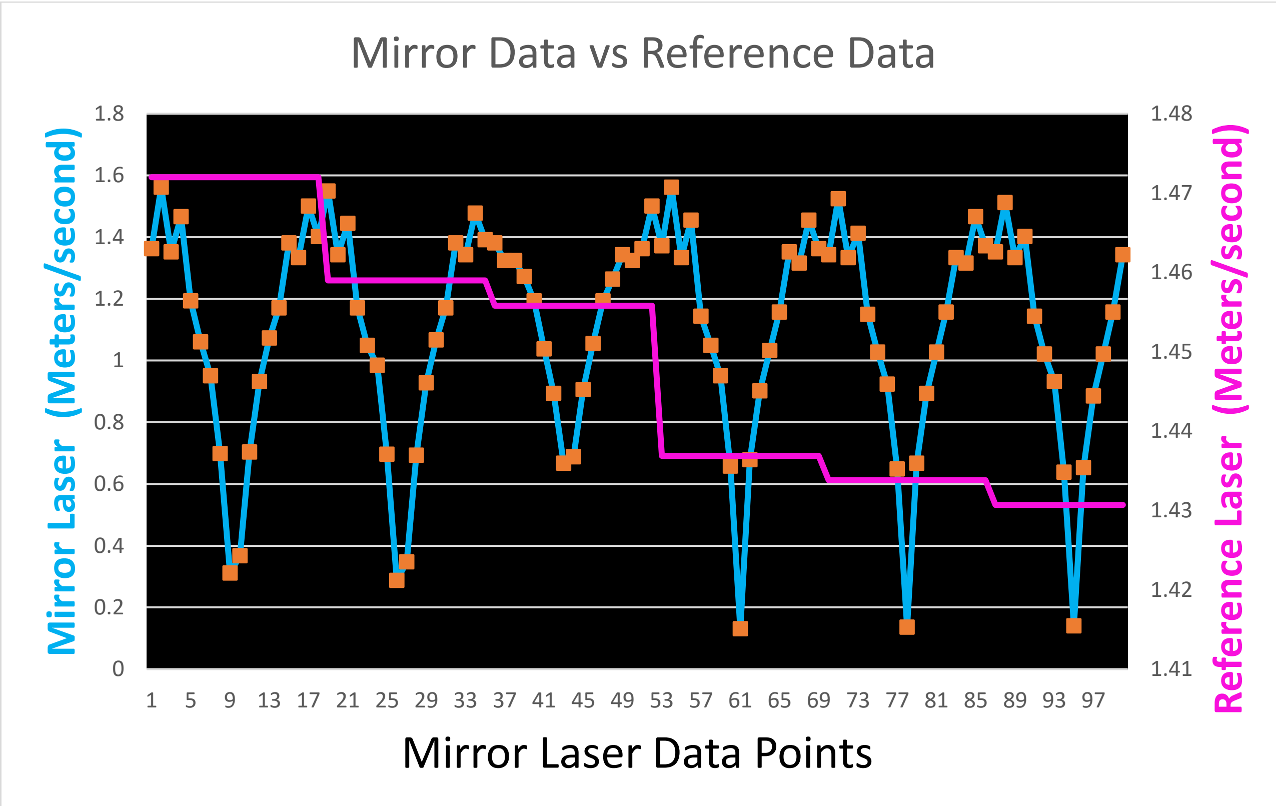


The red trace is the data. The white dotted trace is a best fit exponential decay curve. The formula of the exponential decay is in blue bottom left of chart.

I have found discrepancies in the angle of swing compared to what I measured using 5-degree increment marks on the pendulum assemble of up to 3-5 degrees of error less than the expected. My theory is my ability to measure the distances with a tape measure and the width of the pendulum rod was not highly accurate.

Mirror Laser Data

The mirror laser data is collected in records, with each record of pulses starting when the reference laser beam is broken or twice per swing. Here is a sample of the mirror laser data and the reference laser data combined for comparison:



The speed extrapolated from the mirror laser is in blue, the speed extrapolated from the reference laser is in pink. Note that the reference laser is updated each cycle of the mirror laser as shown, thus we see a step-down pattern as the pendulum loses speed on each swing. Also note that the as we pass the vertical the reference laser shows close to maximum speed and as we approach the extremities of the swing the mirror laser shows the speed decreasing towards zero.

CONCLUSIONS

With the exception of the error found in the angle of swing the data matches what was expected from the theoretical formulas for the pendulum. The mirror data matches what you would expect with speed falling off to zero near the extremities of motion and being maximum at the center of the swing. The period matched very well with the expected period for the pendulum length per the approximation formula. I have really enjoyed this experiment as it validates the theory of pendulum operations to an actual real one, I hope you enjoyed the article and it inspires my fellow scientific minded, curiosity driven friends to run some experiments of their own.

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