# Euler's Formula And The Derivation of Trigonometric Identities

Euler's formula is used throughout engineering, physics and mathematics. Richard Feynman called it "our jewel" and "the most remarkable formula in mathematics". The formula relates complex number with trigonometry. The formula:

$$e^{ix} = \cos(x) + isin(x)$$

This relates the exponential of an angle 'x' given in radians and its equality to its real and imaginary parts. My goal in this article is to show how easily two of the most famous trigonometric identities arise from this formula specifically the following identities.

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a)$$

Many other trigonometric identities can be derived from Euler's formula. To understand this derivation, it must be understood that the formula holds for complex values of 'x' as well as real ones. Consider the following:

$$e^{(a+b)} = e^{a}e^{b}$$

Then it naturally follows that:

$$e^{i(a+b)} = \cos(a+b) + isin(a+b)$$

It also follows that:

$$e^{ia}e^{ib} = (\cos(a) + isin(a))(\cos(b) + isin(b))$$

Performing the multiplication, we get:

$$e^{ia}e^{ib} = \cos(\mathrm{a})\cos(\mathrm{b}) - \sin(\mathrm{a})\sin(\mathrm{b}) + \mathrm{i}(\sin(\mathrm{a})\cos(\mathrm{b}) + \sin(\mathrm{b})\cos(\mathrm{a}))$$

Re-arranging the left side of the equation gives us:

$$e^{i(a+b)} = \cos(\mathrm{a})\cos(\mathrm{b}) - \sin(\mathrm{a})\sin(\mathrm{b}) + \mathrm{i}(\sin(\mathrm{a})\cos(\mathrm{b}) + \sin(\mathrm{b})\cos(\mathrm{a}))$$

But the left is also equal to:

$$e^{i(a+b)} = \cos(a+b) + isin(a+b)$$

This is the same form, notice the real part is separated from the imaginary part or:

$$e^{i(a+b)} = \underline{\cos(\mathrm{a})\cos(\mathrm{b}) - \sin(\mathrm{a})\sin(\mathrm{b})} + \mathrm{i}\underline{(\sin(\mathrm{a})\cos(\mathrm{b}) + \sin(\mathrm{b})\cos(\mathrm{a}))}$$

IS EQUAL TO ↓ IS EQUAL TO ↓

$$e^{i(a+b)} = \cos(a+b) + isin(a+b)$$

These are the trigonometric identities that we set out to confirm. Hope you enjoyed.